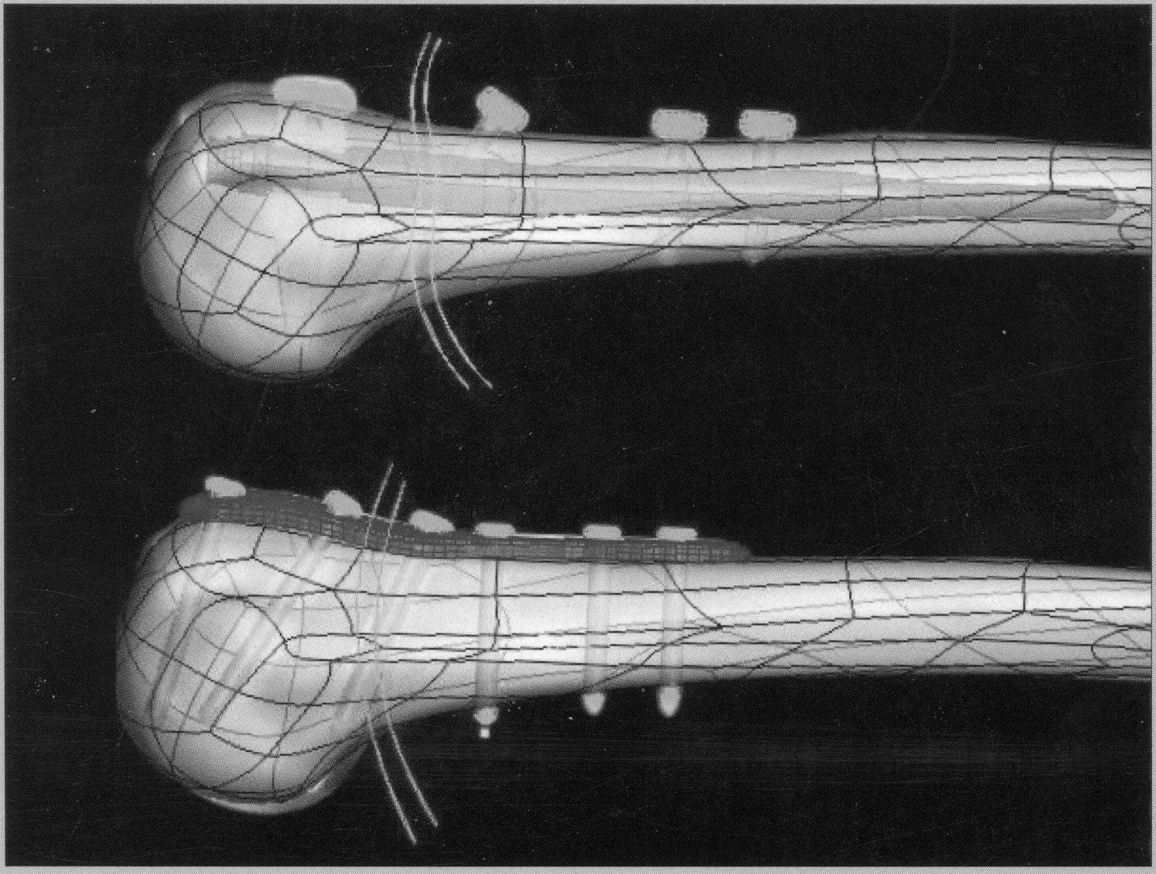
Proceedings of the Fifth International Conference on Engineering Computational Technology

Edited by
B.H.V. Topping, G. Montero and R. Montenegro



Las Palmas de Gran Canaria - Spain
12-15 September 2006

CIVIL-COMP PRESS

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published 2006 by
Civil-Comp Press
Dun Eaglais, Kippen
Stirlingshire, FK8 3DY, UK

Civil-Comp Press is an imprint of Civil-Comp Ltd

ISBN-10 1-905088-09-4 (Book)
ISBN-10 1-905088-10-8 (CD-Rom)
ISBN-10 1-905088-11-6 (Combined Set)

ISBN-13 978-1-905088-09-6 (Book)
ISBN-13 978-1-905088-10-2 (CD-Rom)
ISBN-13 978-1-905088-11-9 (Combined Set)

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

Cover Image: Three-dimensional CAD models of plates and nails used in the treatment of a proximal humeral fracture. This image is used with the permission of C. Pereira. For more details, see Paper 189.

Printed in Great Britain by Bell & Bain Ltd, Glasgow

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Paper 163

A BEM-FEM Model for Studying Dynamic Impedances of Piles in Elastic Soils

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Keywords: boundary element, finite element, BEM-FEM coupling, piles, pile groups, pile-soil interaction, dynamic impedances.

Foundations subject to strong static or dynamic loads and with high responsibility such as those of bridges, off-shore structures, support-walls, nuclear power plants or high buildings, are often solved using piles arranged in groups. Loads arising, for example, from the action of the wind, running machinery or sea waves are usually applied to a rigid cap connecting the top of the piles, so the action is distributed among them. Nevertheless, the different piles in a group will not generally support the same forces, unless the distance between them is big enough, which is not common. This interaction between the piles through the soil medium has been taken into account from early works. This way, the pile group impedances are usually strongly dependent on frequency and its behaviour varies with pile spacing, pile geometry, group size and soil and pile properties.

There are many publications related to this problem (see e.g. [1,2]). Two of the authors of the present work have developed a boundary element model for the computation of dynamic impedances of piles in elastic and poroelastic soils [3,4].

In this work a boundary element - finite element model is presented for the computation of time harmonic dynamic stiffness coeficients of piles embedded in an elastic halfspace. Piles are modelled using finite elements (FEM) as a beam according to the Bernoulli hypothesis, while the soil is modelled using boundary elements (BEM) as a continuum, semi-infinite, isotropic, homogeneous, linear, viscoelastic medium (the semi-infinite soil and radiation damping is easily represenseted by the BEM).

The dynamic model presented is based on previous static model developed by Matos Filho *et al.* [5], where it is assumed that the elastic soil is not disturbed by the piles and the tractions in the pile-soil interface are considered as a load applied within the half-space in the boundary integral representation of the soil.

In the present study, piles are modelled by the FEM as vertical beams according to the Bernoulli hypothesis, and are discretized using three-nodes elements with 13 degrees of freedom defined: two lateral displacements and a vertical displacement on each node, and two rotations on each one of the extreme nodes. Lateral displacements along the element are approximated by a set of fourth degree shape functions, while vertical displacements and tractions along the pile-soil interface are approximated by one of second degree. The sub-matrix that transforms nodal force components to equivalent nodal forces, and the stiffness and mass sub-matrices are defined.

The soil is modelled by the BEM as a linear homogeneous isotropic elastic un-bounded region. Generally, body forces are considered to be zero in elastodynamic problems. Nevertheless, in this case, the tractions within the soil along the pile-soil interface can be treated as loads applied within the half-space, as it is assumed that the soil continuity is not altered by the presence of the pile. The boundary element discretization of the half-space is made using nine-node quadrilateral or six-node triangular elements. The coupling BEM-FEM is imposed by compatibility and equilibrium conditions between the variables of the two methods along the pile shaft.

The main advantage of this model is the capacity of computing accurately stiffness coefficients with low computing times and low memory requirements in comparison to other methods that need to discretize the pile surface or volume. This way, pile groups with a big number of members can be analyzed without difficulty. Besides, once the surface (not necessarily flat) has been discretized, it has not to be changed to analyze different sets of piles, which can be modified easily. Other internal variables such as stress values along the pile can be obtained, and soil strata and rigid rocky beds can be easily taken into account. Furthermore, the model can be included into an existing BEM code by adding subroutines to obtain the mono-dimensional or surface integrals along the pile-soil interface, and modifying the system of equations in the way that is presented.

Several results are presented and compared to well known values taken from the literature, obtaining an excellent agreement.

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Paper 163

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A BEM-FEM Model for Studying Dynamic Impedances of Piles in Elastic Soils

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Abstract

This paper shows a BEM-FEM coupling model for the dynamic analysis of piles and pile groups embedded in an elastic half-space. Piles are modelled using Finite Elements (FEM) as a beam according to the Bernoulli hypothesis, while the soil is modelled using Boundary Elements (BEM) as a continuum, semi-infinite, isotropic, homogeneous, linear, viscoelastic medium. It is assumed that the soil is not disturbed by the piles, and the tractions at the pile-soil interface are considered as a load applied within the half-space. Finally, in order to validate the model, selected numerical results will be presented and compared with other reference values taken from the literature.

Keywords: boundary element, finite element, BEM-FEM coupling, piles, pile groups, pile-soil interaction, dynamic impedances.

1 Introduction

Foundations submitted to strong static or dynamic loads and high responsibility such as those of bridges, off-shore structures, support-walls, nuclear power plants or high buildings, are often solved using piles arranged in groups. Loads arising, for example, from the action of the wind, running machinery or sea waves are usually applied on a rigid cap connecting the top of the piles, so the action is distributed among them. Nevertheless, the different piles in a group will not generally support the same forces, unless the distance between them is big enough, which is not common. This interaction between the piles through the soil medium has been taken into account from early works.

This way, the pile group impedances are usually strongly dependent on frequency and its behaviour varies with pile spacing, pile geometry, group size and soil and pile properties.

This problem, involving dynamic load-displacement analysis of piles and pile groups has received considerable attention during the last few decades. Quite a number of papers have appeared that address the problem using either computational [1–11], rigorous [12–14] or simplified analytical [15–19] techniques. A good compilation of used procedures are the ones presented by Novak [20] or Beskos [21].

An efficient and accurate approach to soil-structure interaction problems is the one regarding BEM (Boundary Elements Method) - FEM (Finite Element Method) coupling, taking advantage of the particular characteristics of each. During the last years, great progress has been made on this topic [22–24], some of which dealing with pile-soil interaction. For instance, Code and Venturini [25] present the coupling of framed structures approached by FEM with three-dimensional bodies represented by BEM in time domain, where piles would be approximated using a special cylindrical boundary element.

However, a different BEM-FEM model for the computation of time harmonic dynamic stiffness coefficients of piles groups embedded in an elastic half-space is presented in this work, where piles are modelled using Finite Elements (FEM) as a beam according to the Bernoulli hypothesis, while the soil is modelled using Boundary Elements (BEM) as a continuum, semi-infinite, isotropic, homogeneous, linear, viscoelastic medium. The dynamic model presented is based on previous static model developed by Matos Filho et al [26], where it is assumed that the elastic soil is not disturbed by the piles and the tractions in the pile-soil interface are considered as a load applied within the half-space in the boundary integral representation of the soil.

Although the presented results are restricted to a half-space, the technique is very versatile and more complicated problems can be solved. Besides, as the pile boundary does not need to be discretized, low computing times and memory requirements are needed. Selected numerical results for vertical, horizontal and rocking impedances are presented and compared to others taken from the literature.

2 Pile FE equations

The behaviour of a pile submitted to dynamic loads can be described by the following differential equation

$$\mathbf{M}\,\ddot{\mathbf{u}} + \mathbf{C}\,\dot{\mathbf{u}} + \mathbf{K}\,\mathbf{u} = \mathbf{f}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices of the pile, $\mathbf{u}$ is the vector of nodal displacements, $\dot{\mathbf{u}}$ and $\ddot{\mathbf{u}}$ its first and second derivative referring to time, and $\mathbf{f}(t)$ the vector of nodal forces over the pile.

It will be assumed now that the pile is subjected to a harmonically varying load. In this case, the vectors of nodal displacements and forces can be expressed as

$$\mathbf{u} = \bar{\mathbf{u}}^p \, e^{i\omega t} \quad ; \quad \mathbf{f} = \mathbf{F} \, e^{i\omega t} \tag{2}$$

where $\bar{\mathbf{u}}^p$ is the vector of nodal displacements and turns amplitudes, $\mathbf{F}$ is the vector of nodal forces amplitudes and ω the circular frequency of the excitement. Then, and considering a pile with zero internal damping, equation (1) becomes

$$(\mathbf{K} - \omega^2 \mathbf{M}) \, \bar{\mathbf{u}}^p = \mathbf{F} \tag{3}$$

Piles are modelled by FEM as vertical beams according to the Bernoulli hypothesis, and are discretized using the three-nodes element that is shown in Figure 1. There are 13 degrees of freedom defined on it: two lateral displacements and a vertical displacement on each node, and two rotations θ on each one of the extreme nodes, one about x_1 axis and another one about x_2.

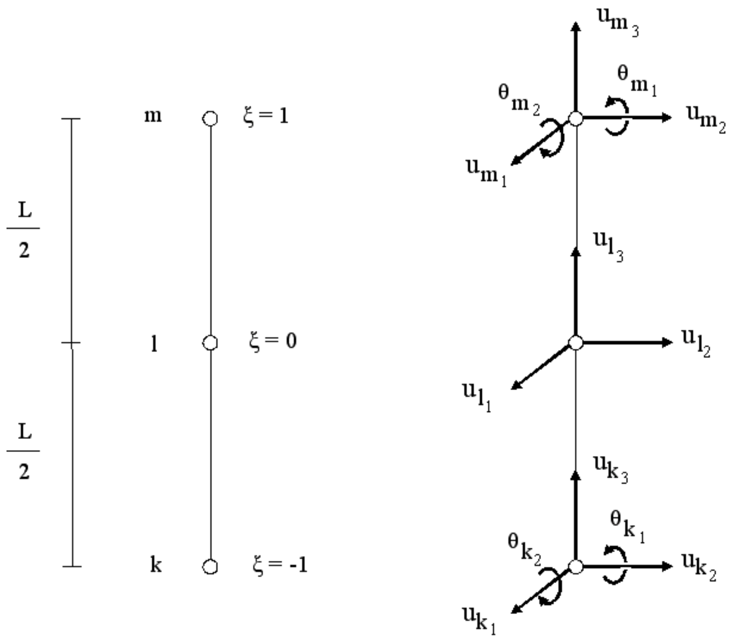

Figure 1: Element definition

The lateral displacements u_1 and u_2 along the element are approximated by a set of fourth degree shape functions, while vertical displacements u_3 are approximated by one of second degree. Thus

$$u_i = \varphi_{k1} u_{k_i} + \varphi_{k2} \theta_{k_i} + \varphi_l u_{l_i} + \varphi_{m1} u_{m_i} + \varphi_{m2} \theta_{m_i} \tag{4}$$

$$u_3 = \phi_k u_{k_3} + \phi_l u_{l_3} + \phi_m u_{m_3} \tag{5}$$

where

$$
\begin{aligned}
\varphi_{k_1} &= \xi(-\frac{3}{4} + \xi + \frac{1}{4}\xi^2 - \frac{1}{2}\xi^3) \\
\varphi_{k_2} &= \frac{1}{4}\xi(-1 + \xi + \xi^2 - \xi^3) \\
\varphi_l &= 1 - 2\xi^2 + \xi^4 \\
\varphi_{m_1} &= \xi(\frac{3}{4} + \xi - \frac{1}{4}\xi^2 - \frac{1}{2}\xi^3) \\
\varphi_{m_2} &= \frac{1}{4}\xi(-1 - \xi + \xi^2 + \xi^3)
\end{aligned}
\tag{6}
$$

and

$$
\begin{aligned}
\phi_k &= \frac{1}{2}\xi(\xi - 1) \\
\phi_l &= 1 - \xi^2 \\
\phi_m &= \frac{1}{2}\xi(\xi + 1)
\end{aligned}
\tag{7}
$$

where ξ is the elemental dimensionless coordinate varying from -1 to $+1$.

Using the principle of virtual displacements and the shape functions defined above, the stiffness sub-matrix for the lateral behaviour of this element can be obtained as (see reference [27])

$$k^l_{ij} = \int_L \varphi''_i EI \varphi''_j \, dx \tag{8}$$

and the one for the axial behaviour as

$$k^a_{ij} = \int_L \phi'_i EA \phi'_j \, dx \tag{9}$$

where *E* is the Young's Modulus for the pile, *A* and *I* are the area and the moment of inertia of the section of the pile and *L* is the element length.

Finally, the matrices obtained are

$$
\begin{pmatrix} f_{k_i} \\ M_{k_i} \\ f_{l_i} \\ f_{m_i} \\ M_{m_i} \end{pmatrix}
= \frac{EI}{5L}
\begin{bmatrix}
\frac{316}{L^2} & \frac{94}{L} & \frac{-512}{L^2} & \frac{196}{L^2} & \frac{-34}{L} \\
\frac{94}{L} & 36 & \frac{-128}{L} & \frac{34}{L} & -6 \\
\frac{512}{L^2} & \frac{-128}{L} & \frac{1024}{L^2} & \frac{-512}{L^2} & \frac{128}{L} \\
\frac{196}{L^2} & \frac{34}{L} & \frac{-512}{L^2} & \frac{316}{L^2} & \frac{-94}{L} \\
\frac{-34}{L} & -6 & \frac{-128}{L} & \frac{-94}{L} & 36
\end{bmatrix}
\begin{pmatrix} u_{k_i} \\ \theta_{k_i} \\ u_{l_i} \\ u_{m_i} \\ \theta_{m_i} \end{pmatrix}
\quad ; \quad i = 1, 2 \tag{10}
$$

and

$$\begin{pmatrix} f_{k_3} \\ f_{l_3} \\ f_{m_3} \end{pmatrix} = \frac{EA}{3L} \begin{bmatrix} 7 & -8 & 1 \\ -8 & 16 & -8 \\ 1 & -8 & 7 \end{bmatrix} \begin{pmatrix} u_{k_3} \\ u_{l_3} \\ u_{m_3} \end{pmatrix} \tag{11}$$

Similarly, the mass influence coefficients for an element, that represents the inertia force opposing the acceleration experimented by a certain degree of freedom, can be evaluated by a similar procedure as

$$m_{ij} = \int_L \psi_i m \psi_j \, dx \tag{12}$$

Using the same functions that were used for calculating the stiffness matrix, the result obtained is the consistent-mass matrix. Thus, considering a beam with uniformly distributed mass *m*, the matrices obtained for the lateral and axial behaviours are, respectively

$$\mathbf{M}^l = Lm \begin{bmatrix} \frac{13}{63} & \frac{L}{63} & \frac{4}{63} & \frac{-23}{630} & \frac{L}{180} \\ \frac{L}{63} & \frac{L^2}{630} & \frac{2L}{315} & \frac{-L}{180} & \frac{L^2}{1260} \\ \frac{4}{63} & \frac{2L}{315} & \frac{128}{315} & \frac{4}{63} & \frac{-2L}{315} \\ \frac{-23}{630} & \frac{-L}{180} & \frac{4}{63} & \frac{13}{63} & \frac{-L}{63} \\ \frac{L}{180} & \frac{L^2}{1260} & \frac{-2L}{315} & \frac{-L}{63} & \frac{L^2}{630} \end{bmatrix} \tag{13}$$

and

$$\mathbf{M}^a = \frac{Lm}{15} \begin{bmatrix} 2 & 1 & \frac{-1}{2} \\ 1 & 8 & 1 \\ \frac{-1}{2} & 1 & 2 \end{bmatrix} \tag{14}$$

The vector of nodal forces **F** can be decomposed as

$$\mathbf{F} = \mathbf{F}^{ext} + \mathbf{F}^{eq} \tag{15}$$

where $\mathbf{F}^{ext}$ are the forces at the top of the pile and $\mathbf{F}^{eq}$ is the vector of the equivalent nodal forces from the pile-soil interaction, that can be calculated as

$$\mathbf{F}^{eq} = \mathbf{Q} \cdot \mathbf{q}^p \tag{16}$$

where **Q** is the matrix that transforms nodal force components to equivalent nodal forces.

As shown in figure 2, the tractions $\mathbf{q}^p$ along the pile-soil interface are approximated by the set of shape functions defined by equation (7) as

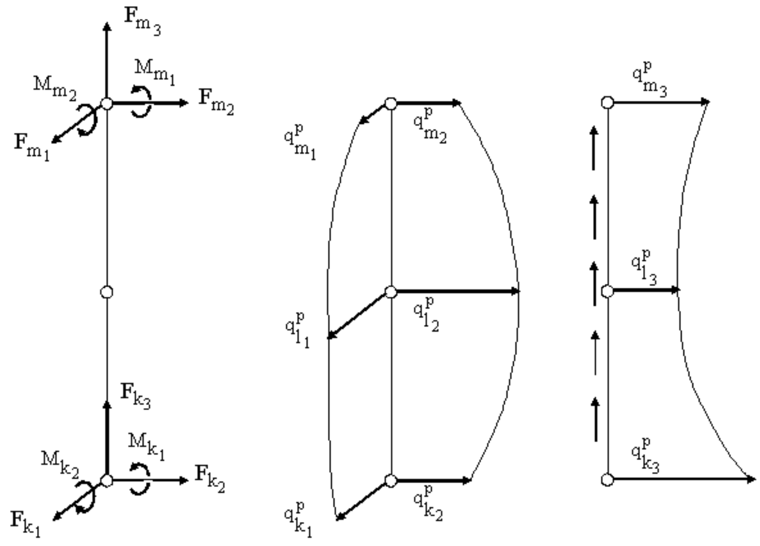

Figure 2: External forces and tractions along the pile-soil interface

$$q_i = \phi_k q_{k_i} + \phi_l q_{l_i} + \phi_m q_{m_i} \tag{17}$$

Again, using the principle of virtual displacements, the coefficients of matrix **Q** for lateral forces can be obtained as

$$q^l_{ij} = \int_L \varphi_i \phi_j \, dx \tag{18}$$

and the ones for axial forces as

$$q^a_{ij} = \int_L \phi_i \phi_j \, dx \tag{19}$$

This way, one can obtain the following matrices for lateral and axial equivalent nodal forces respectively

$$\begin{pmatrix} f^{eq}_{k_i} \\ M^{eq}_{k_i} \\ f^{eq}_{l_i} \\ f^{eq}_{m_i} \\ M^{eq}_{m_i} \end{pmatrix} = \begin{bmatrix} \frac{23L}{140} & \frac{11L}{105} & \frac{-L}{28} \\ \frac{L^2}{84} & \frac{L^2}{105} & \frac{-L^2}{210} \\ \frac{4L}{105} & \frac{16L}{35} & \frac{4L}{105} \\ \frac{-L}{28} & \frac{11L}{105} & \frac{23L}{140} \\ \frac{L^2}{210} & \frac{-L^2}{105} & \frac{-L^2}{84} \end{bmatrix} \begin{pmatrix} q^p_{k_i} \\ q^p_{l_i} \\ q^p_{m_i} \end{pmatrix} \quad ; \quad i = 1, 2 \tag{20}$$

and

$$\left(\begin{array}{c} f^{eq}_{k_3} \\ f^{eq}_{l_3} \\ f^{eq}_{m_3} \end{array} \right) = \frac{L}{30} \left[\begin{array}{ccc} 4 & 2 & -1 \\ 2 & 16 & 2 \\ -1 & 2 & 4 \end{array} \right] \left(\begin{array}{c} q^{p}_{k_3} \\ q^{p}_{l_3} \\ q^{p}_{m_3} \end{array} \right) \tag{21}$$

Once all elemental matrices have been obtained, one can write for the whole pile

$$\bar{\mathbf{K}}\,\bar{\mathbf{u}}^{p} = \mathbf{F}^{ext} + \mathbf{Q}\,\mathbf{q}^{p} \tag{22}$$

where $\bar{\mathbf{K}} = \mathbf{K} - \omega^2\mathbf{M}$. As each pile will be discretized using as many elements as necessary to follow its deformed shape accurately, matrices $\bar{\mathbf{K}}$ and $\mathbf{Q}$ are global matrices, obtained as usual from the elemental ones.

3 Soil BE equations

The soil is modelled by BEM as a linear homogeneous isotropic elastic un-bounded region. The boundary integral equation for a time-harmonic elastodynamic state defined in the domain Ω with boundary Γ can be written in a condensed and general form as

$$\mathbf{c}^k\mathbf{u}^k + \int_{\Gamma} \mathbf{p}^*\mathbf{u}\,d\Gamma = \int_{\Gamma} \mathbf{u}^*\mathbf{p}\,d\Gamma + \int_{\Omega} \mathbf{u}^*\mathbf{X}\,d\Omega \tag{23}$$

where $\mathbf{u}$ and $\mathbf{p}$ are the displacements and tractions vectors

$$\mathbf{u} = \left\{ \begin{array}{c} u_1 \\ u_2 \\ u_3 \end{array} \right\} \quad \mathbf{p} = \left\{ \begin{array}{c} t_1 \\ t_2 \\ t_3 \end{array} \right\} \tag{24}$$

$\mathbf{u}^*$ and $\mathbf{p}^*$ are the elastodynamic fundamental solution tensors on the boundary Γ due to a unit load concentrated at point 'k'

$$\mathbf{u}^* = \left[\begin{array}{ccc} u^*_{11} & u^*_{12} & u^*_{13} \\ u^*_{21} & u^*_{22} & u^*_{23} \\ u^*_{31} & u^*_{32} & u^*_{33} \end{array} \right] \quad \mathbf{p}^* = \left[\begin{array}{ccc} t^*_{11} & t^*_{12} & t^*_{13} \\ t^*_{21} & t^*_{22} & t^*_{23} \\ t^*_{31} & t^*_{32} & t^*_{33} \end{array} \right] \tag{25}$$

$\mathbf{c}^k$ is the local free term matrix at collocation point $\mathbf{x}_k$ with the form

$$\mathbf{c}^k = \begin{cases} \mathbf{I}, & \text{for internal points} \\ 0.5\,\mathbf{I}, & \text{for boundary points where the boundary is smooth} \end{cases} \tag{26}$$

$\mathbf{I}$ being the unit 3×3 diagonal matrix, and $\mathbf{X}$ are the body forces in the domain Ω

Generally, these body forces are considered to be zero in elastodynamic problems. Nevertheless, in this case, the tractions $\mathbf{q}^{s_j}$ within the soil along the j^{th} pile-soil interface can be treated as loads applied within the half-space, as it is assumed that the soil continuity is not altered by the presence of the pile. Then, equation (23) can be written as

$$\mathbf{c}^k\mathbf{u}^k + \int_\Gamma \mathbf{p}^*\mathbf{u}\, d\Gamma = \int_\Gamma \mathbf{u}^*\mathbf{p}\, d\Gamma + \sum_{j=1}^{n_p} \int_{\Gamma_{p_j}} \mathbf{u}^*\mathbf{q}^{s_j}\, d\Gamma_{p_j} \tag{27}$$

where Γ_{p_j} is the pile-soil interface of pile j and n_p is the total number of piles.

Equation (27) for a boundary point is calculated numerically. To do so, the boundary surface is discretized into quadratic elements of triangular and quadrilateral shape with six and nine nodes, respectively (see reference [28]) and using quadratic shape functions. These functions are used to represent the boundary variables and the geometry.

Once the boundary has been discretized and the unit load applied on all nodes in Γ, equation (27) can be written in matrix form as

$$\mathbf{H}^{ss}\mathbf{u}^s = \mathbf{G}^{ss}\mathbf{p} + \sum_{j=1}^{n_p} \mathbf{G}^{sp_j}\mathbf{q}^{s_j} \tag{28}$$

where $\mathbf{u}^s$ is the vector of nodal displacements on the surface, $\mathbf{H}^{ss}$ and $\mathbf{G}^{ss}$ are matrices obtained by integration over Γ of the 3-D elastodynamic fundamental solution times the shape functions of the boundary elements, and $\mathbf{G}^{sp_j}$ is the matrix obtained by integration over Γ_{p_j} of the 3-D elastodynamic fundamental solution times the interpolation functions defined in (7), when the unit load is applied over Γ. Assuming free traction surface ($\mathbf{p} = 0$), equation (28) becomes

$$\mathbf{H}^{ss}\mathbf{u}^s - \sum_{j=1}^{n_p} \mathbf{G}^{sp_j}\mathbf{q}^{s_j} = 0 \tag{29}$$

Furthermore, the unit load will be also applied on the pile nodes. The top node must be treated as a surface node on a smooth boundary and the rest of them as internal points. Then, applying equation (27) over a certain pile named i, one can write

$$\mathbf{H}^{p_i s}\mathbf{u}^s - \sum_{j=1}^{n_p} \mathbf{G}^{p_i p_j}\mathbf{q}^{s_j} + \mathbf{C}\,\mathbf{u}_k^{p_i} = 0 \tag{30}$$

where $\mathbf{u}_k^{p_i}$ is the vector of nodal displacements at the node k of the pile i where the unit load is applied, $\mathbf{H}^{p_i s}$ is the matrix obtained by integration over Γ of the 3-D elastodynamic fundamental solution times the shape functions of the boundary elements, and $\mathbf{G}^{p_i p_j}$ is the matrix obtained by integration over Γ_{p_j} of the 3-D elastodynamic fundamental solution times the interpolation functions defined in (7), when the unit load is applied over a pile i. $\mathbf{C}$ is a diagonal matrix with a 1.0 on rows and columns corresponding to internal points and a 0.5 on rows and columns corresponding to top nodes.

The integrals over Γ are solved numerically according to reference [29], while integrals over Γ_{p_j} are calculated either as an integral extended over a cylinder which radius is that of the pile, when the collocation point is within it, or as a monodimensional integral extended to a load-line, when the collocation point is outside it. In such a case, this line is defined by the pile axis.

4 BEM-FEM coupling equation

Now, a global system of equation must be built using the expressions defined above. The links between piles and soil that will allow us to do the coupling are the tractions $\mathbf{q}$ along the pile-soil interface and the displacements $\mathbf{u}^p$ along the pile. Using equilibrium and compatibility conditions along the interface, and assuming the tractions $\mathbf{q}^s$ as positive, equations (22), (29) and (30) can be rearranged as

$$\mathbf{A}\,\mathbf{x} = \mathbf{B} \tag{31}$$

where $\mathbf{B}$ is the right-hand vector when all external conditions have been applied,

$$\mathbf{A} = \begin{bmatrix} \mathbf{H}^{ss} & -\mathbf{G}^{sp_1} & -\mathbf{G}^{sp_2} & \cdots & -\mathbf{G}^{sp_{n_p}} & \emptyset & \emptyset & \cdots & \emptyset \\ \mathbf{H}^{p_1 s} & -\mathbf{G}^{p_1 p_1} & -\mathbf{G}^{p_1 p_2} & \cdots & -\mathbf{G}^{p_1 p_{n_p}} & \mathbf{C}'_{p_1} & \emptyset & \cdots & \emptyset \\ \mathbf{H}^{p_2 s} & -\mathbf{G}^{p_2 p_1} & -\mathbf{G}^{p_2 p_2} & \cdots & -\mathbf{G}^{p_2 p_{n_p}} & \emptyset & \mathbf{C}'_{p_2} & \cdots & \emptyset \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & & \vdots \\ \mathbf{H}^{p_{n_p} s} & -\mathbf{G}^{p_{n_p} p_1} & -\mathbf{G}^{p_{n_p} p_2} & \cdots & -\mathbf{G}^{p_{n_p} p_{n_p}} & \emptyset & \emptyset & \cdots & \mathbf{C}'_{p_{n_p}} \\ \emptyset & \mathbf{Q}^{p_1} & \emptyset & \cdots & \emptyset & \bar{\mathbf{K}}^{p_1} & \emptyset & \cdots & \emptyset \\ \emptyset & \emptyset & \mathbf{Q}^{p_2} & \cdots & \emptyset & \emptyset & \bar{\mathbf{K}}^{p_2} & \cdots & \emptyset \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \ddots & \vdots \\ \emptyset & \emptyset & \emptyset & \cdots & \mathbf{Q}^{p_{n_p}} & \emptyset & \emptyset & \cdots & \bar{\mathbf{K}}^{p_{n_p}} \end{bmatrix} \tag{32}$$

and

$$\mathbf{x} = \{\mathbf{u}^s, \mathbf{q}^{s_1}, \mathbf{q}^{s_2}, \ldots, \mathbf{q}^{s_{n_p}}, \bar{\mathbf{u}}^{p_1}, \bar{\mathbf{u}}^{p_2}, \ldots, \bar{\mathbf{u}}^{p_{n_p}}\}^T \tag{33}$$

5 Dynamic stiffness of piles and pile groups

The dynamic stiffness matrix K_{ij} of a pile relates to the vector of forces (and moments) applied at the pile top and the resulting vector of displacements (and rotations) at the same point. For a group of piles, it is assumed that the pile heads are constrained by a rigid pile-cap, and the foundation stiffness is the addition of the contributions of each pile. Fig 3 illustrates the approached problem for a usual configuration, where *L* and *d* are used to denote the length and diameter of the piles, and *s* refers to the distance between adjacent piles.

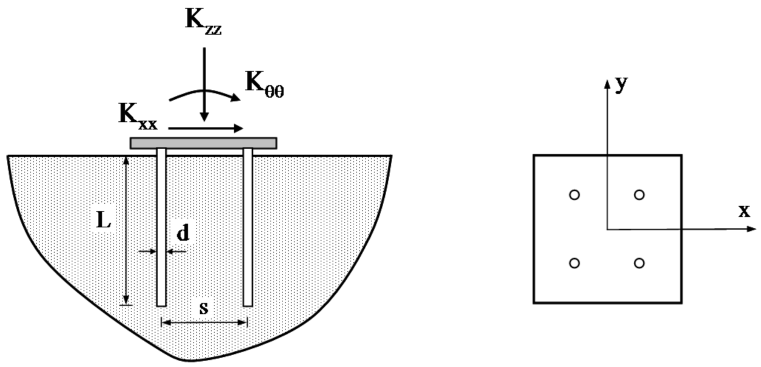

Figure 3: 2×2 pile group embedded in a half-space. Problem geometry definition.

The dynamic stiffness terms for a time harmonic excitation are functions of frequency ω and they are usually written as

$$K_{ij} = k_{ij} + \mathrm{i} a_o c_{ij} \tag{34}$$

where k_{ij} and c_{ij} are the frequency dependent dynamic stiffness and damping coefficients, respectively, c_s is the soil shear-wave velocity and a_o is the dimensionless frequency

$$a_o = \frac{\omega d}{c_s} \tag{35}$$

6 Validation and numerical results

In order to validate the MEC-MEF coupling model, several results of impedances of piles and groups of piles are contrasted with other reference values taken from the literature.

Figure 4 shows a sketch of the discretizations used to obtain the stiffness of different pile groups embedded in a viscoelastic half-space, where boundary elements for the soil and mono-dimensional finite elements for the piles were used. As the developed software incorporates symmetry properties, only a quarter of the total geometry of the problem has to be discretized. Rectangular quadratic nine-nodes elements were used on the surface. The length of free surface needed (that has to be discretized because there is not any global fundamental solution of easy implementation for time-harmonic elastic problems) is found through experiments, searching for the convergence of the solution, and the element size is chosen in such a way that its main dimension is always shorter than the half of the wave length. On the other hand, the three-nodes elements defined above were used on the pile. To get an accurate solution, each pile had to be discretized using only three elements for vertical problems, fifteen elements for rocking problems, and five elements for horizontal problems.

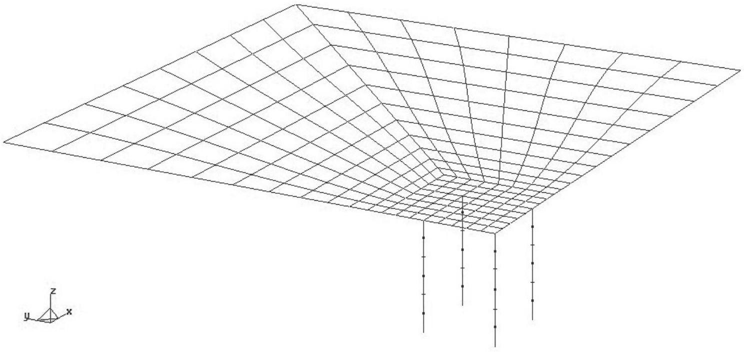

Figure 4: A quarter of the free surface and pile discretization for a 3x3 pile group in a half-space

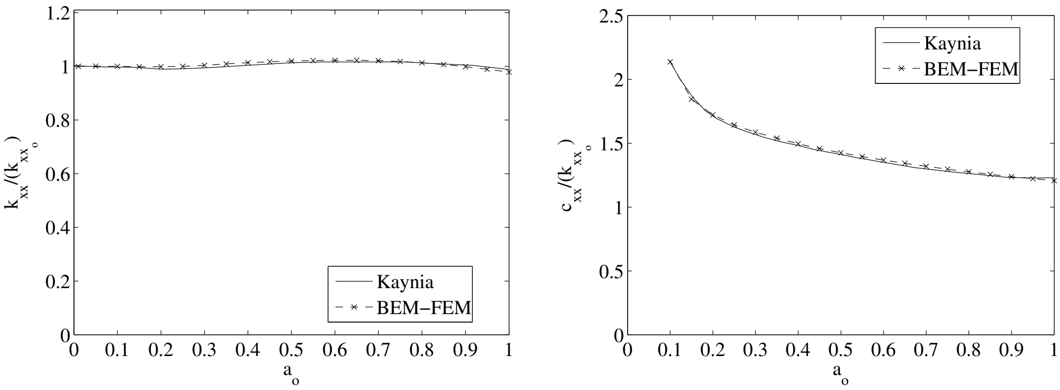

Figure 5: Horizontal impedances of a single pile. Comparison with Kaynia's solution.

Lateral and vertical impedances (real and imaginary parts) of single piles embedded in a homogeneous isotropic viscoelastic half-space, obtained by proposed technique (noted by BEM-FEM in the figures), are shown in figures 5 and 6. The lateral and rocking impedances of 2x2 and 3x3 pile groups are shown from figure 7 to figure 10, and vertical impedances 4x4 pile groups are shown in figures 11, all for $L/d = 2, 5, 10$. All these results are compared with those of Kaynia and Kausel [2], obtained from an analysis of single piles and pile groups considering piles as linear elastic prismatic members and soil as semi-infinite viscoelastic media by constructing the requisite Green's function using a discrete layer matrix approach.

The following properties are taken from Kaynia and Kausel [2]: piles (in the sequel denoted by sub-index p) are assumed to be elastic Bernoulli beams; and surrounding soil a uniform viscoelastic media with internal damping coefficient $\beta = 0.05$; the ratio between the material modulae is $E_p/E = 10^3$; ratio between densities $\rho/\rho_p = 0.7$; and Poisson's ratios $\nu = 0.4$ (for the soil) and $\nu_p = 0.25$ (for the pile, but not taken into account in the proposed technique). The piles aspect ratio is $L/d = 15$. The vertical and horizontal impedance functions for several spacing to diameter ratios L/d have

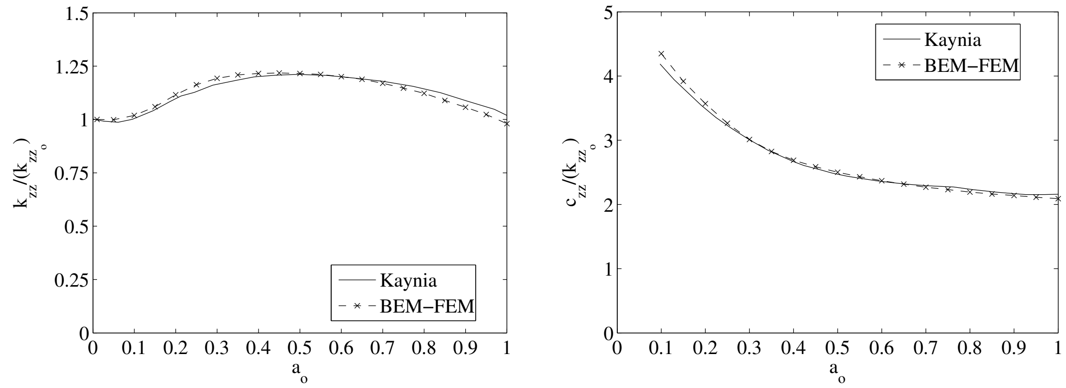

Figure 6: Vertical impedances of a single pile. Comparison with Kaynia's solution.

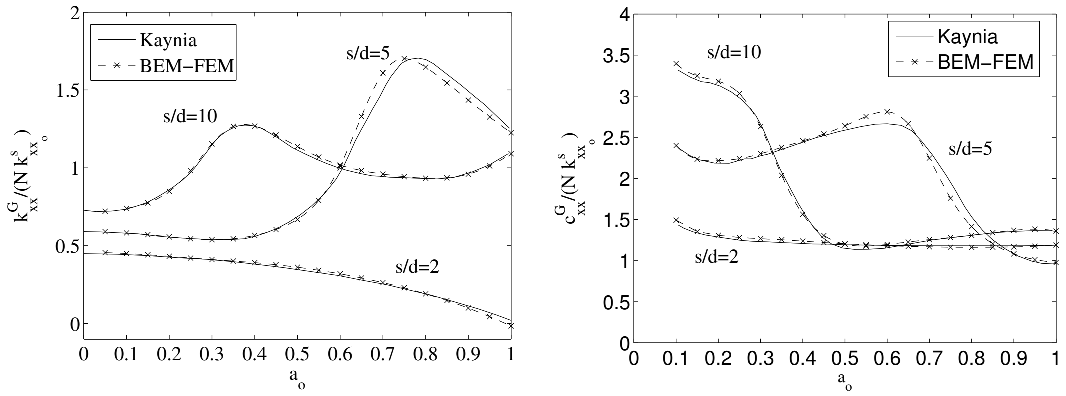

Figure 7: Horizontal impedances of 2x2 pile groups. Comparison with Kaynia's solution.

been normalized with respect to the respective single pile static stiffness (k^s) times the number (N) of piles in the group. The rocking impedances have been normalized with respect to the sum of the products of the respective single pile static stiffness (k^s_{zz}) times the square of the distance to the rotation axe (x_i). Besides, all results are plotted against the dimensionless frequency parameter defined by equation (35).

It can be seen that the computed values are in very good agreement with those presented in [2].

7 Revision and conclusion

In this paper, a three-dimensional BEM-FEM coupling model for the computation of time-harmonic dynamic stiffness coefficients of piles and pile groups embedded in a homogeneous isotropic viscoelastic soils has been presented. Piles are modelled using Finite Elements (FEM) as a beam according to the Bernoulli hypothesis, while the soil

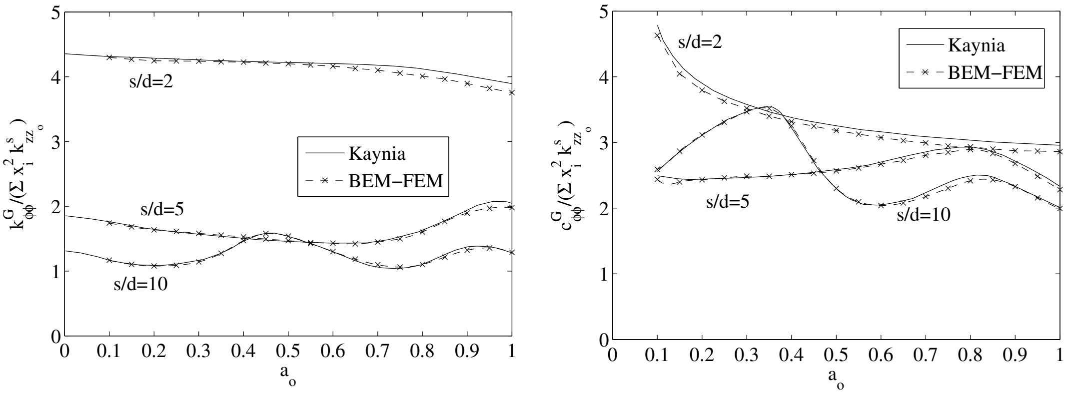

Figure 8: Rocking impedances of 2x2 pile groups. Comparison with Kaynia's solution.

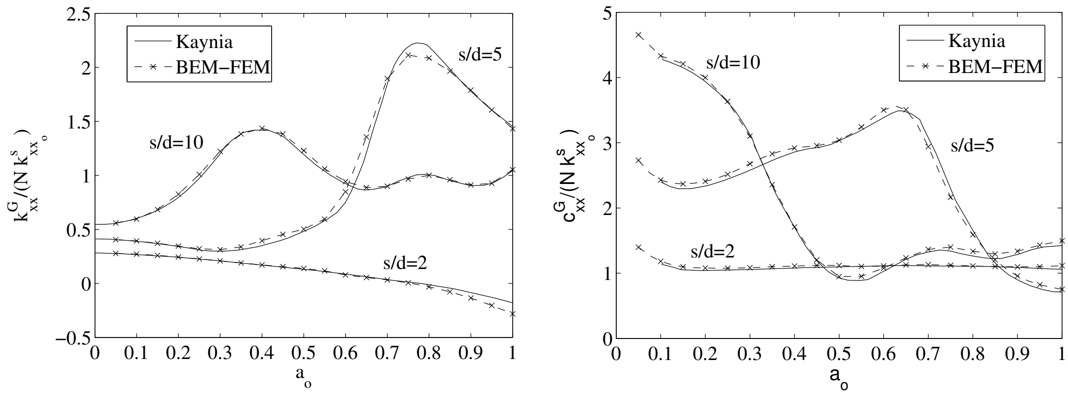

Figure 9: Horizontal impedances of 3x3 pile groups. Comparison with Kaynia's solution.

is modelled using Boundary Elements (BEM) as a continuum, semi-infinite, isotropic, homogeneous, linear, viscoelastic medium.

The main advantage of this model is the capacity of computing accurately stiffness coefficients with low computing times and low memory requirements in comparison to other methods that need to discretize the pile surface or volume. This way, pile groups with a big number of members can be analyzed without difficulty. Besides, once the surface (not necessarily flat) has been discretized, it has not to be changed to analyze different sets of piles, which can be modified easily. Other internal variables such as stress values along the pile can be obtained, and soil strata and rigid rocky beds can be easily taken into account. Furthermore, the model can be included into an existing BEM code by adding subroutines to obtain the mono-dimensional or surface integrals along the pile-soil interface, and modifying the system of equations in the way that has been presented above.

Several results have been presented and compared to well known values taken from

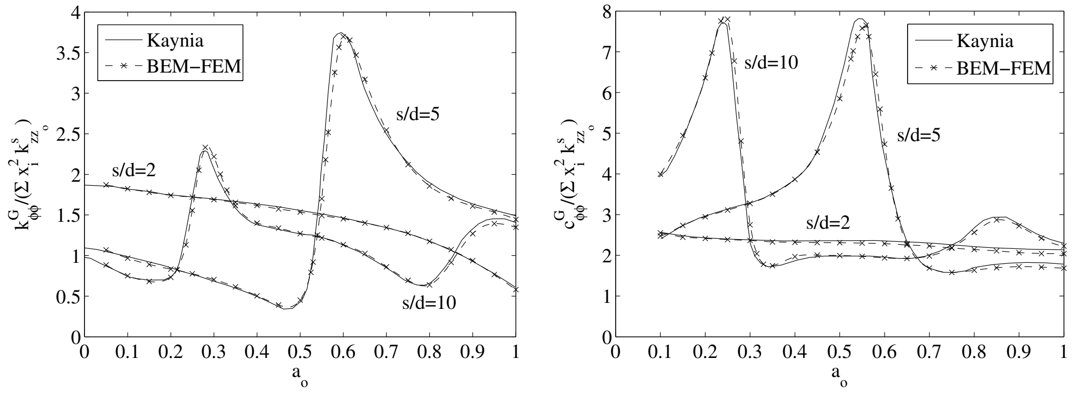

Figure 10: Rocking impedances of 3x3 pile groups. Comparison with Kaynia's solution.

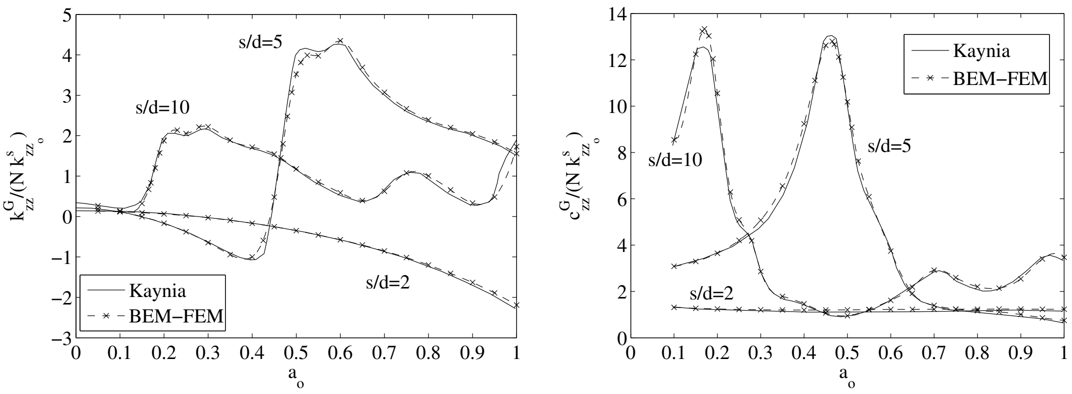

Figure 11: Vertical impedances of 4x4 pile groups. Comparison with Kaynia's solution.

the literature, obtaining an excellent agreement. More cases than these presented in this work have been tested, all of them with the favorable conclusions.

Future developments that are being considered are the generalization of the model to include: isotropic homogeneous fluid-filled poroelastic soils governed by Biot's theory, steady time-harmonic plane waves coming from the far field, flexible raft foundations and super-structures coupling.

Acknowledgements

The work presented in this paper was partially supported by the Ministry of Education and Science of Spain through research project BIA2004-03955-C02-02. Also, L. Padrón is recipient of the FPU research fellowship AP-2004-4858 from the Ministry of Education and Science of Spain. Authors would like to thank for this support.

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